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Anomalies and supersymmetry breaking

F Delduc† and G M Shore‡

† TH Division, CERN, Geneva-1211, Switzerland

‡ Blackett Laboratory, Imperial College of Science and Technology, Prince Consort Road, London SW7 2BZ, UK

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Abstract. The particle spectrum of a spontaneously broken supersymmetric gauge theory may in principle contain massive vector bosons without the scalars required for complete vector supermultiplets (supersymmetry 'shattering' or breaking 'in spins'). It is conjectured, however, that this truncation of the spectrum is incompatible with gauge anomaly freedom, and thus cannot occur in a unitary quantum field theory. A supersymmetric Higgs model is presented illustrating clearly the role of anomalies in ensuring an untruncated spectrum of supersymmetry multiplets modulo mass splittings, and providing strong evidence in favour of the conjecture.

1. Introduction

In this paper we investigate the particle spectrum in a spontaneously broken supersymmetric gauge theory. In particular, we conjecture that the conditions for anomaly freedom are sufficient to preclude the unusual pattern of supersymmetry and gauge breaking found in [1], where the massive vector bosons arising from the Higgs mechanism are *not* accompanied by the scalars which would be required to form complete massive vector supermultiplets.

In a supersymmetric theory, the Goldstone bosons of a spontaneously broken internal symmetry lie in massless chiral multiplets. It is possible for both the scalars in a chiral multiplet to be themselves Goldstone bosons—they are then said to be 'non-doubled'. If only one of the scalars is a Goldstone boson, it is 'doubled'. The simplest form of the supersymmetric Higgs mechanism occurs when a massless gauge vector multiplet (2_B+2_F degrees of freedom) couples to a doubled Goldstone multiplet (2_B+2_F). The Goldstone boson combines with the massless vector to produce a massive vector boson (3_B), leaving a massive scalar (1_B), while the two Weyl fermions combine into a massive Dirac fermion (4_F). This is the content of a massive vector supermultiplet. However, if the Goldstone multiplet is non-doubled, then *two* massless gauge vector multiplets can couple to it. After the Higgs mechanism, the spectrum in this case comprises two massive vector bosons ($2\times 3_B$), *no* scalars, a massive Dirac fermion (4_F) and a massless Weyl fermion (2_F). This spectrum does not have the required number of degrees of freedom to form complete massive vector supermultiplets—the spectrum is truncated. This form of spontaneous supersymmetry breaking has been called breaking 'in spins' [2], or, more evocatively, supersymmetry 'shattering' [1] (see also the review [3]).

An explicit model, with gauge group $SU(2)\times U(1)_Y$ breaking to $U(1)_{em}$, which displays this effect was presented in [1]. However, this model contained just one SU(2) doublet chiral superfield and so was anomalous. As shown in [4], removing the anomalies by adding a second doublet with opposite hypercharge leads to doubled Goldstone multiplets and a conventional non-truncated spectrum. Several other models, also in the context of dynamical gauge symmetry breaking [5], share this property.

This leads us to make the following conjecture† (see also [6]) that *in an anomaly-free theory, supersymmetry 'shattering' is not possible.* This intimate relation between gauge anomalies and supersymmetry breaking 'in spins' is an intriguing and possibly deep result. It is of course purely quantum mechanical—at the semiclassical level there is no incompatibility.

The purpose of this paper is to sharpen the evidence for this conjecture, and to investigate whether it is feasible to promote it to a no-go theorem. We first formulate a set of necessary conditions for breaking to a truncated spectrum to occur, then construct a model in which as many as possible of these conditions are realised. Despite this, the model—a chiral gauge theory with gauge group $SU(5)\times U(1)$—still fails to produce a truncated spectrum. It shows very clearly the role of gauge anomaly freedom, and in our view provides very strong evidence that supersymmetry shattering is impossible to achieve.

2. Necessary conditions for supersymmetry shattering

The general formalism describing the supersymmetric Higgs mechanism in weak coupling gauge theories is given in [1, 4], and so here we shall simply quote results as needed. Two general theorems from [1] are worth recalling.

(i) It is not possible for all Goldstone multiplets to be non-doubled. There must exist at least one doubled multiplet.

(ii) If gauge fields are coupled to both the scalars in a non-doubled Goldstone multiplet, then supersymmetry is spontaneously broken by the D-type mechanism. For this to occur it is necessary for the gauge group to have a U(1) factor with a Fayet–Iliopoulos term [8]. We therefore consider models with gauge group $G_W = G\times U(1)$.

We now formulate a set of necessary conditions for the occurrence of supersymmetry shattering in a unitary quantum field theory. If this is indeed impossible, we might hope to formulate a no-go theorem by showing the inconsistency of these conditions (or, even better, of a subset of them). We shall see later to what extent this hope is borne out.

For simplicity, we consider models with no superpotential (in the light of the results from the model in § 3 it seems unlikely that this would help). The effective potential is then

$$V=\tfrac{1}{2}(D^A)^2 \tag{1}$$

with

$$D^A=-g\sum_{(r)}\varphi^*_{(r)}T^A_{(r)}\varphi_{(r)} \qquad A\neq 0 \tag{2}$$

$$D^0=-g'\sum_{(r)}\varphi^*_{(r)}T^0_{(r)}\varphi_{(r)}+2\zeta \tag{3}$$

† The difficulties of obtaining supersymmetry shattering in an anomaly-free model have been noted independently by W Lerche (private communication). Related work on symmetry breaking patterns in supersymmetric theories may be found in [7].

where $T^0_{(r)}\varphi_{(r)} = Q_Y(r)\varphi_{(r)}$, and $T^A_{(r)}$ are the generators of G in the representation (r). The U(1) generator is denoted by the index 0 and ζ is the Fayet-Iliopoulos coefficient. We have used the equations of motion for the gauge auxiliary fields D^A in writing (1).

The vacuum expectation values must be such that the potential is extremised, i.e. $\partial V/\partial\varphi|_{\langle\varphi\rangle} = 0$. This implies

$$g^A\langle D^A\rangle T^A_{(r)}\langle\varphi_{(r)}\rangle = 0. \tag{4}$$

In the case where supersymmetry is spontaneously broken, $(\langle D^A\rangle \neq 0)$, (4) states that the generator $\langle D^A\rangle T^A$ remains unbroken. Moreover, it may be shown to commute with all other conserved generators and thus generate an unbroken U(1).

Further, the potential must be minimised. This means that the (squared) mass matrix of the bosons must not contain any negative eigenvalues. The expression for the mass matrix may be found in [4]. Unfortunately, this condition is rather complicated to incorporate as a necessary ingredient of a possible no-go theorem.

A necessary condition for obtaining a truncated spectrum is that some gauge fields couple to non-doubled Goldstone bosons. The criterion for non-doubling [1, 9] is that there exists a complex linear combination $c_A T^A$ of generators which is broken, but for which the complex combination $c^*_A T^A$ is unbroken, i.e.

$$\begin{aligned} &c_A T^A_{(r)}\langle\varphi_{(r)}\rangle \neq 0 &&\text{for some representation } (r)\\ &c^*_A T^A_{(r)}\langle\varphi_{(r)}\rangle = 0 &&\text{for all representations } (r). \end{aligned} \tag{5}$$

Finally, we impose the anomaly freedom conditions, which we believe to preclude supersymmetry shattering even when condition (5) is fulfilled:

$$\sum_{(r)} A(r) = 0 \tag{6a}$$

where the 'anomaly' $A(r)$ of the representation (r) is defined by $\mathrm{Tr}\, T^A_{(r)}\{T^B_{(r)}, T^C_{(r)}\} = A(r)\, d^{ABC}$ $(A, B, C \neq 0)$,

$$\sum_{(r)} I(r) Q_Y(r) = 0 \tag{6b}$$

where the index $I(r)$ of the representation (r) is defined by $\mathrm{Tr}\{T^B_{(r)}, T^C_{(r)}\} = I(r)\delta^{AB}$, and

$$\sum_{(r)} D(r) Q_Y(r)^3 = 0 \tag{6c}$$

where $D(r)$ is the dimension of (r).

3. The SU(5) × U(1) model

We now present a model which is constructed so as to have the best possible chance of realising supersymmetry shattering. The model is a chiral gauge theory, since a chirally asymmetric field content clearly enhances the possibility of obtaining non-doubled Goldstone bosons after spontaneous symmetry breaking. It also has the property that, in the absence of the Fayet-Iliopoulos term, the only minimum is at $\langle\varphi_{(r)}\rangle = 0$. For $\zeta \neq 0$, this excludes the possibility of having a supersymmetric vacuum with all $\langle D^A\rangle = 0$.

We consider† a gauge group $SU(5)\times U(1)_Y$ and ensure the absence of anomalies by choosing a field content corresponding to the reduction of the **16** of SO(10) with respect to $SU(5)\times U(1)_Y$‡§

	SU(5)	$U(1)_Y$	$A(r)$	$I(r)$
S_i	5	3	1	1
M^{ij}	$\overline{10}$	−1	−1	3
P	1	−5	0	0

It is easily shown that the most general form of the vacuum expectation values is, up to an $SU(5)\times U(1)_Y$ transformation,

$$\langle S\rangle=\begin{pmatrix}0\\0\\0\\0\\v\end{pmatrix}\qquad \langle M\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0&u_1&0&0&0\\-u_1&0&u_2&0&0\\0&-u_2&0&u_3&0\\0&0&-u_3&0&u_4\\0&0&0&-u_4&0\end{pmatrix}\qquad \langle P\rangle=w\,\mathrm{e}^{\mathrm{i}\alpha} \tag{7}$$

The corresponding vacuum expectation values of the D^A fields are then computed using (2) and (3). The *a priori* non-vanishing $\langle D^A\rangle$ correspond to the diagonal generators T^0, $T^1=\mathrm{diag}(-1,1,0,0,0)$, $T^2=(1/\sqrt{3})\,\mathrm{diag}(-1,-1,2,0,0)$, $T^3=(1/\sqrt{6})$ $\mathrm{diag}(-1,-1,-1,3,0)$, $T^4=(1/\sqrt{10})\,\mathrm{diag}(-1,-1,-1,-1,4)$ and to the non-diagonal generators $(T_{kl})_{ij}=\delta_{ki}\delta_{lj}+\delta_{li}\delta_{kj}$ for $(k,l)=(1,3)$, $(2,4)$ and $(3,5)$. The vacuum energy is then

$$\begin{aligned}\langle V\rangle &= g'^2(u_1^2+u_2^2+u_3^2+u_4^2-3v^2+w^2+2\zeta/g')^2\\ &\quad+(g^2/10)(-2u_1^2-2u_2^2-2u_3^2+3u_4^2-4v^2)^2\\ &\quad+(g^2/6)(-2u_1^2-2u_2^2+2u_3^2+3u_4^2)^2\\ &\quad+g^2u_2^4+(g^2/3)(-2u_1^2+u_2^2+2u_3^2)^2\\ &\quad+4g^2(u_1^2u_2^2+u_2^2u_3^2+u_3^2u_4^2).\end{aligned} \tag{8}$$

The extremum of the potential with highest symmetry corresponds to $v\neq 0$, $u_i=w=0$, and the unbroken gauge group is $SU(4)\times U(1)$ (we take $\zeta>0$). The decomposition

† Without the $U(1)_Y$ gauging and the chiral superfield P, this model is the one-flavour version of that considered in [10] to illustrate supersymmetry breaking by instantons. Here, however, we gauge $U(1)_Y$ and add a Fayet–Iliopoulos term, so that the fields acquire non-trivial VEV at the tree level and the model may be described as a weak coupling theory with spontaneously broken gauge symmetry (of course, since there are scalars in the fundamental representation of SU(5), complementarity implies there is an equivalent description in terms of unbroken gauge symmetry). Instanton effects are therefore negligible in determining the spectrum.

‡ This choice of fields also ensures that the trace of the $U(1)_Y$ charge vanishes, which in turn implies the vanishing of the Abelian gravitational anomaly [11]. The vanishing of the $U(1)_Y$ trace is also necessary for the D term not to be renormalised [12].

§ In fact, the model has three classical U(1) symmetries [10]: $U(1)_Y$, which is anomaly free and commutes with supersymmetry; $U(1)_R$, which is anomaly free but does not commute with supersymmetry; and $U(1)_A$ which commutes with supersymmetry but is not anomaly free. Thus only $U(1)_Y$ is suitable for gauging. Both $U(1)_R$ and $U(1)_A$ are spontaneously broken here in the preferred vacuum.

of the representations is as follows:

$$\begin{aligned}
\mathrm{SU}(5)\times \mathrm{U}(1) &\to \mathrm{SU}(4)\times \mathrm{U}(1)\\
5(3) &\to 4(3)+1(0)\\
\overline{10}(-1) &\to 6(-2)+\bar{4}(1)\\
1(-5) &\to 1(-4).
\end{aligned}$$

If the $\overline{10}$ representation is not included in the model, this would be a minimum of the potential and an example of a model with non-doubling and a truncated spectrum. There are four non-doubled Goldstone multiplets and one doubled, and, as explained in the introduction, massive vector bosons arise without the necessary additional states required for complete massive vector supermultiplets. However, it can be shown that, for the irreducible representations of $\mathrm{SU}(5)\times \mathrm{U}(1)$ whose vacuum expectation value vanishes, the squared mass after symmetry breaking is proportional to the opposite of the unbroken U(1) charge. Thus the $\bar{4}$ coming from the decomposition of the $\overline{10}$ would have a negative squared mass and this extremum is not a minimum. The symmetry has to break further.

It can in fact be shown that there is a minimum with $v\neq 0$, $u_4\neq 0$, all other $u_i=0$, and $w=0$ (this means that the $\bar{4}$ of SU(4) has acquired a vacuum expectation value). The unbroken gauge group is $\mathrm{SU}(3)\times \mathrm{U}(1)$, and the decomposition of the representation is as follows:

$$\begin{aligned}
\mathrm{SU}(5)\times \mathrm{U}(1) &\to \mathrm{SU}(3)\times \mathrm{U}(1)\\
5(3) &\to 3(2)+1(3)+1(0)\\
\overline{10}(-1) &\to 3(-1)+\bar{3}(-2)+\bar{3}(1)+1(0)\\
1(-5) &\to 1(-3).
\end{aligned}$$

Notice that the spectrum obtained is chirally symmetric with respect to the unbroken subgroup. To achieve this breaking, sixteen Goldstone bosons are needed. These are eaten in the Higgs mechanism to produce the massive vector bosons. We have sixteen chiral multiplets in the triplet and singlet representations specified. However, not all of these contain Goldstone bosons. In fact, we still have non-doubling.

The sixteen broken generators decompose under $\mathrm{SU}(3)\times \mathrm{U}(1)$ into $[3(-1)+\bar{3}(1)]+[3(2)+\bar{3}(-2)]+[1(3)+1(-3)]+1(0)+1(0)$. It is easy to check that the following complex linear combinations of SU(5) generators satisfy the non-doubling criterion (6), viz $c_A T^A_{ij}=\delta_{ki}\delta_{4j}$ for $k=1,2,3$ and $c_A T_{ij}=\delta_{4i}\delta_{5j}$. These generators $c_A T^A_{ij}$ are broken whereas $c^*_A T^A_{ij}$ are not. They correspond to the $\mathrm{SU}(3)\times \mathrm{U}(1)$ representation $\bar{3}(1)+1(3)$. As a consequence, the Goldstone multiplets split into four non-doubled multiplets, namely $\bar{3}(1)+1(3)$, and eight doubled multiplets, namely $3(2)+\bar{3}(-2)+1(0)+1(0)$.

Remarkably therefore, in this model, *all* the conditions of § 2 are satisfied. The mechanism [1] described in the introduction where two massless vector multiplets combine with just one chiral Goldstone multiplet takes place. However, there are additional chiral multiplets not containing Goldstone bosons with exactly the right quantum numbers to complete massive vector supermultiplets. These are the $3(-1)$ coming from the $\overline{10}$ and the $1(-3)$ coming from the singlet. These *spectator* fields, which take no part in the Higgs mechanism, provide the extra degrees of freedom needed to produce a non-truncated spectrum†. The presence of these spectators is

† Notice that these spectator fields will automatically be present if the spectrum of chiral superfields is a self-adjoint representation of the unbroken subgroup.

required by the anomaly freedom conditions. All the sixteen original fields *S*, *M* and *P* are therefore used up, so the final spectrum of the model comprises simply the massless gauge vector multiplets of $SU(3)\times U(1)$ and, modulo mass splittings, the sixteen massive gauge vector multiplets of $SU(5)\times U(1)/SU(3)\times U(1)$.

Notice, however, that in the above discussion we have only considered the quantum numbers of the particles in the physical spectrum. It remains to be shown whether the supersymmetry transformations can indeed be modified so as to relate the states in the incomplete massive vector multiplets and those in the spectator chiral multiplets.

4. Conclusion and discussion

In the particular model presented, we have shown that anomaly freedom forces the introduction of spectator chiral multiplets with precisely the right quantum numbers needed for complete vector multiplets in what would otherwise be a truncated spectrum. We have not proved a direct connection between the absence of anomalies and the impossibility of obtaining a truncated spectrum. However, the evidence from this and many other models points strongly in favour of the conjecture that supersymmetry shattering does not occur in an anomaly-free model. It appears that unitarity of the quantum field theory is incompatible with supersymmetry breaking 'in spins'.

Finally, we have only discussed here the possibility of obtaining supersymmetry shattering in Higgs models. In fact, the effect was suggested [2, 13] on the basis of counting degrees of freedom in gauged supersymmetric non-linear sigma models where the (complex) scalar fields span a homogeneous Kähler manifold G/H. Interestingly, such models have since been shown to possess isometry anomalies [14], and are therefore inconsistent when gauged. The anomaly-free sigma models which actually arise as low-energy effective Lagrangians from a fundamental supersymmetric theory with spontaneous chiral symmetry breaking have as target manifold the deformation of a complex coset space $\bar{G}/\tilde{K}$ where the metric possesses only G isometries (G being the compact global symmetry group of the fundamental theory and $\bar{G}$ its complexification) [15, 16]. In such models, the chiral symmetry breaking pattern $G\to H$ depends on the *embedding* of the complex isotropy group $\tilde{K}$ in $\bar{G}$. This vacuum alignment problem is resolved when a subgroup of G is gauged [7]. The symmetry breaking $G\to H$ is therefore dependent on the weak gauge field dynamics. It seems likely, therefore, that in sigma models, just as in Higgs models, anomaly freedom precludes supersymmetry breaking 'in spins'.

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